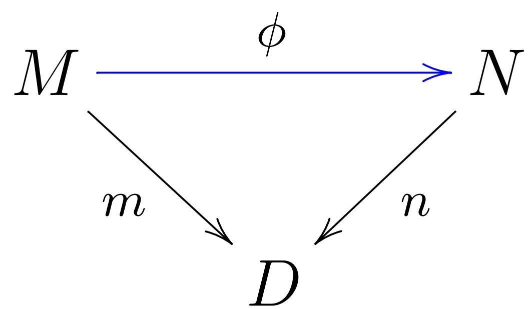
Quiz

Define the subobject of an object D.

Answer:

There are actually two answers:

- A subobject is an equivalance class of monics with target D. Here two monics $m : M \to D$ and $n : N \to D$ are said to be equivalent if there exists a commutative diagram



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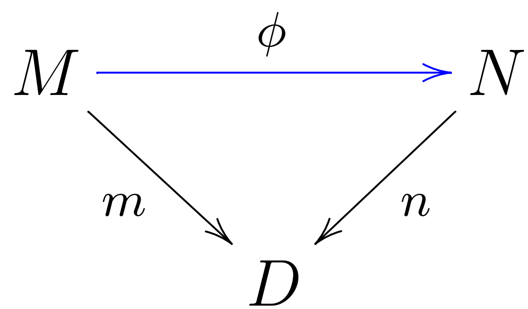


  with ϕ an isomorphism.

- A subobject can be defined too, as a pair $< M, m >$ where $m : M \to D$ is a monic

Last Time:

We proved that every arrow in an Abelian Category can be factored as a composition of a strong epic followed by a monic

This Time:

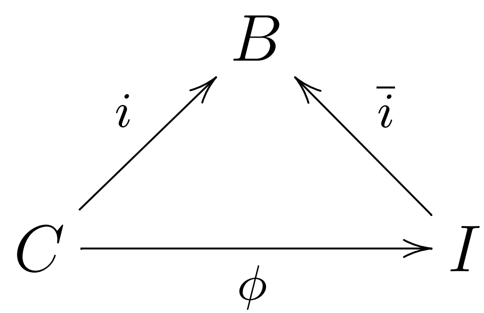
We'll do a few things, but first we will use Monday's proof and a lemma from Chapter 4 to prove the

‘First Isomorphism Theorem”

Let f be an arrow in an Abelian category. Then $im(f) \simeq coim(f)$.

Proof

We have proven already that, $f = i \circ p = \bar{i} \circ \bar{p}$, where i is the image of f and $\bar{i}$ is the coimage. Since i and p (respectively $\bar{p}$) is a kernel (respectively cokernel), i (respectively p) is a strong mono (respectively epi) by and its dual. implies that $i : C \to B$ and $\bar{i} : I \to B$ are isomorphic. That is, there exists an isomorphism ϕ so that the diagram



commutes. In other words, the coimage C and image I are isomorphic.

Definition

Let X be a set. A Equvialence Relation on X is a subset $\sim \subseteq X \times X$ so that for all $x, y, z \in X$:

1. $< x, x > \in \sim$ (Reflexivity)
2. $< x, y > \in \sim$ iff $< y, x > \in \sim$ (Symmetry)
3. $< x, y > \in \sim$ and $< y, z > \in \sim \Rightarrow < x, z > \in \sim$. (Transitivity)

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- The relation $n \sim m$ iff $n - m$ is divisible by 3 is an equivalence relation on the set of integers.
- For fans of Calc III: two directed line segments are equivalent iff they have the same length and direction is an equivalence relation on $\mathbb{R}^3$.

Notation

Let X be a set and $x \in X$. Let $\sim$ be an equivalence relation on X. For ease of notation we will write $x \sim y$ if $< x, y > \in \sim$.

Definition

Let X be a set and $x \in X$. Let $\sim$ be an equivalence relation on X. Then, the equivalence class defined by x is $\{y \in X | x \sim y\}$.

Lemma

Let X be a set. Any equivalence relation on X defines a partition of X, and conversely any partition of X defines an equivalence relation.

Proof

Exercise

Lemma

Let X be an object in a category. Let $j : J \to X$ and $i : I \to X$ be two monics. Let us say that $i \sim j \Rightarrow i \simeq j$. Then, $\sim$ is an equivalence relation.

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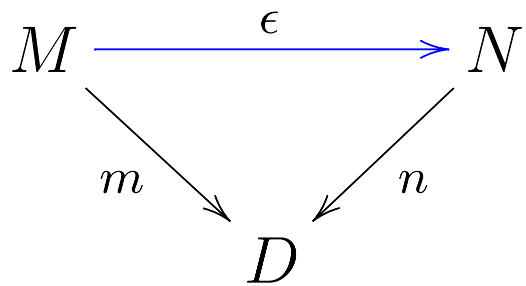
- Symmetry: Suppose $i \sim j$. Then there exists an isomorphism ϕ so that $i = \phi \circ j$. But then $\phi{-}1 \circ i = j$, and $\phi{-}1$ is an isomorphism.
- Transitivity: Suppose $i \sim j$ and $j \sim k$. Then there exists isomorphisms ϕ and ψ so that $i = \phi \circ j$ and $j = \psi \circ k$. This implies though that $i = \phi \circ \psi \circ k$ and since the composition of ϕ and ψ, being the composition of two isomorphisms, is, itself, an isomorphism, we have $i \sim k$.

Let $m : M \to D$ and $n : N \to D$ be two monics. Set $M \leq N$ iff there exists a commutative diagram:



with ϵ monic.

Fact

This relation is a partial ordering on subobjects - regardless of which definition is used.

Next Time:

We will finish §5.3. I plan - for now - to stick to the object, monic definition of subobject.