

Quiz

Define pre-additive

Definition

Any category with Hom sets which meet the criteria listed in the last slide is called **pre-additive**

Consequence

Lemma

Let A and B be objects in \mathbf{C} , a preadditive category. Then, the following are equivalent:

- The product $A \amalg B$ exists

Lemma

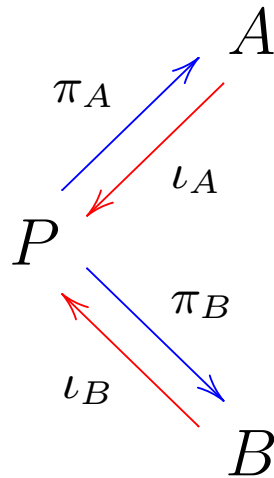
Let A and B be objects in \mathbf{C} , a preadditive category. Then, the following are equivalent:

- The product $A \amalg B$ exists
- The coproduct $A \coprod B$ exists

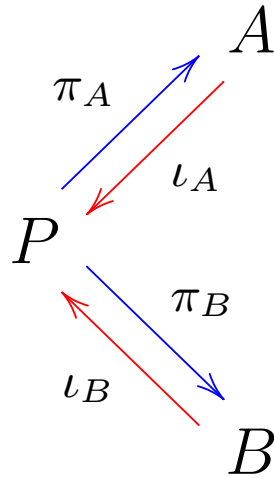
Lemma

Let A and B be objects in \mathbf{C} , a preadditive category. Then, the following are equivalent:

- The product $A \amalg B$ exists
- The coproduct $A \amalg B$ exists
- There exists an object P and arrows

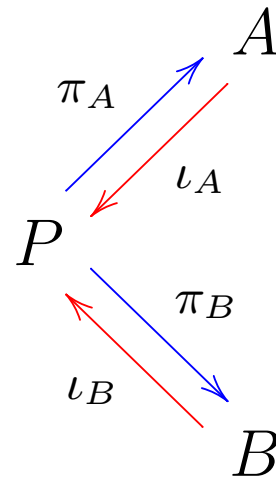


so that



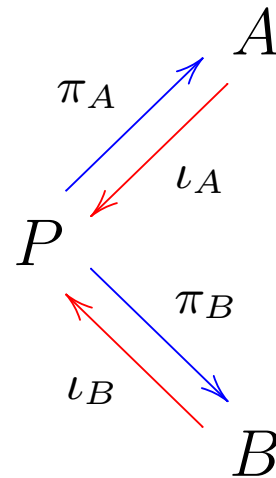
• $\pi_A \circ \iota_A = 1_A$ and $\pi_B \circ \iota_B = 1_B$

so that



- $\pi_A \circ \iota_A = 1_A$ and $\pi_B \circ \iota_B = 1_B$
- $\pi_A \circ \iota_B = 0_{BA}$ and $\pi_B \circ \iota_A = 0_{AB}$

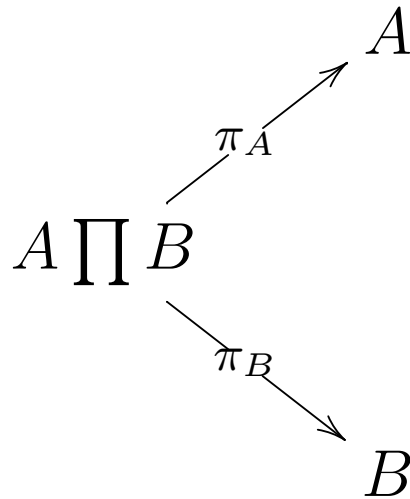
so that



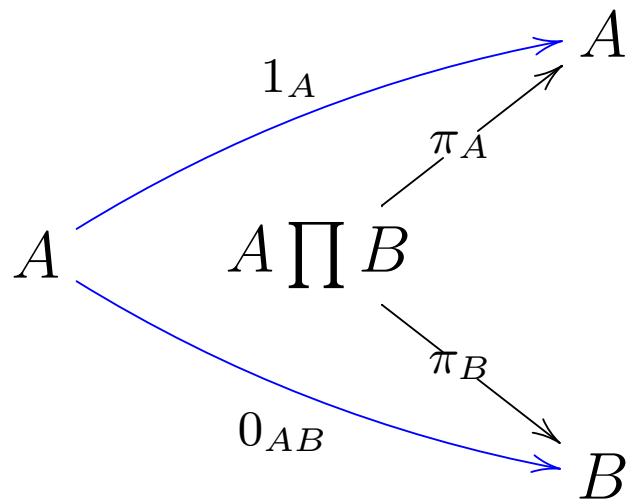
- $\pi_A \circ \iota_A = 1_A$ and $\pi_B \circ \iota_B = 1_B$
- $\pi_A \circ \iota_B = 0_{BA}$ and $\pi_B \circ \iota_A = 0_{AB}$
- $\iota_A \circ \pi_A + \iota_B \circ \pi_B = 1_P$ commutes.

proof

Suppose the product $A \amalg B$ exists. Let

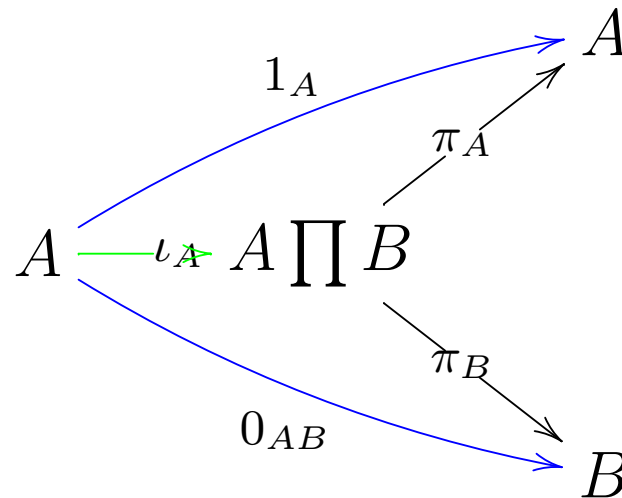


be the product diagram. Consider the diagram

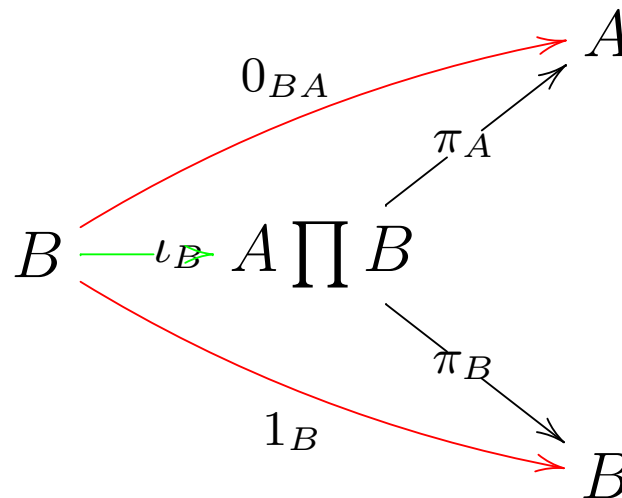


Then there exists

a unique map ι_A so that



commutes. Similarly we have a unique map ι_B so that



commutes

Now,

a short calculation shows that

- $\pi_A \circ (\iota_A \circ \pi_A + \iota_B \circ \pi_B) = \pi_A + 0_{PA} = \pi_A$ and,

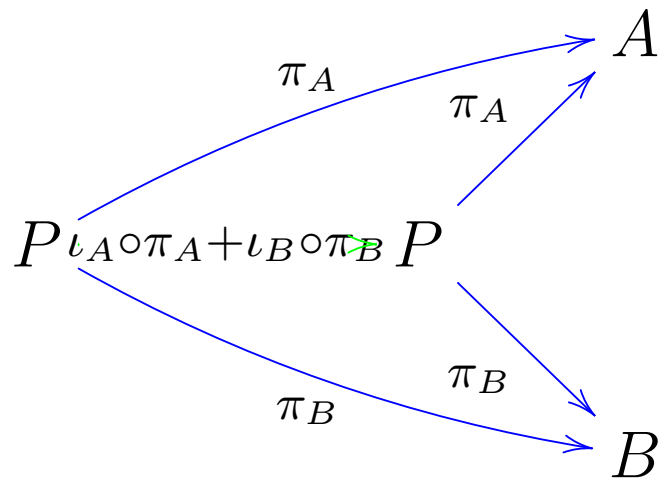
Now,

a short calculation shows that

- $\pi_A \circ (\iota_A \circ \pi_A + \iota_B \circ \pi_B) = \pi_A + 0_{PA} = \pi_A$ and,

- $\pi_B \circ (\iota_A \circ \pi_A + \iota_B \circ \pi_B) = \pi_B + 0_{PB} = \pi_B$

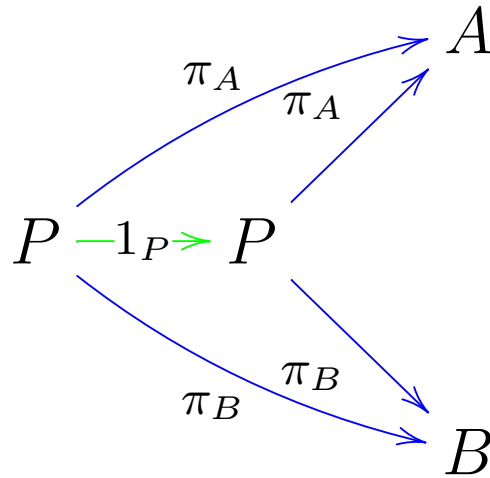
In other words,



commutes.

But,

clearly, 1_P is the unique map which makes



commute. Thus, $\iota_A \circ \pi_A + \iota_B \circ \pi_B = 1_P$.