Quiz

Define the $kernel$ of an arrow f.

Last Time

The End (for the moment) of the prelude

This Time

The first movement:
(Abelian) Groups

The strategy

- List Axioms which define the category of Abelian groups - $\mathbf{AbG}$

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- Prove some classic and some not so classic results using those axioms
- (Tentatively) Examine the internal structure the objects $must$ have if they meet their external structure satisfies the axioms.

We won’t

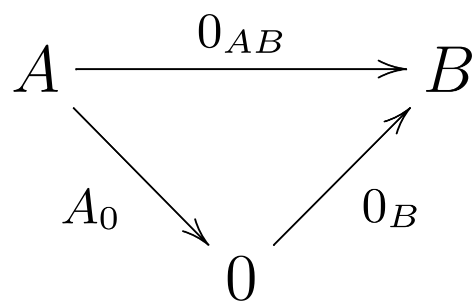
however, necessarily do this in a strictly linear fashion

First Axiom

The category $\mathbf{AbG}$ has an object called the $zero$ object which is simultaneously initial and terminal.

This implies

That for every two objects A and B in $\mathbf{AbG}$, there exists a unique arrow which we will - usually - denote 0_{AB}.



where A_0 is the unique arrow from A to 0 (since 0 is terminal) and where 0_B is the unique arrow from 0 to B. (since 0 is initial)

The Answer

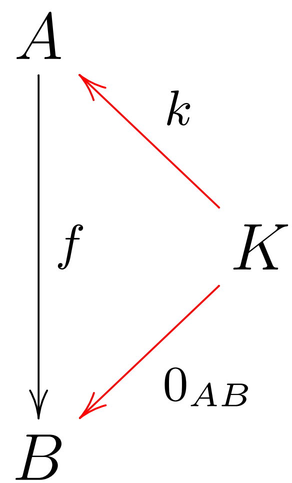
to the quiz:

Definition

Let $f : A \longrightarrow B$ be an arrow in $\mathbf{AbG}$. The *kernel* of f is defined to be the equalizer of f and 0_{AB}.

We will

for the purpose of greater “diagramtic simplicity” often draw the equalizer diagram which describes the kernel $k : K \to A$ of f in this way:



Why

Does this suffice?

Axiom 2

The Hom sets in $\mathbf{AbG}$ have a particular structure. This structure is familiar.

Let A and B and C be objects in $\mathbf{AbG}$. Then, $Hom(A,B)$ is equipped with an assignment $+ : Hom(A,B) \times Hom(A,B) \to Hom(A,B)$ which satisfies the following for every $f, g \in Hom(A,B)$ and $h \in Hom(B,C)$:

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- $f + g = g + f$
- $f + (g + h) = (f + g) + h$
- $f + 0_{AB} = f$
- $h \circ (f + g) = h \circ f + h \circ g$
- There exists an element $\bar{f} \in Hom(A, B)$ so that $f + \bar{f} = 0_{AB}$.