HW §6.1 Numbers 2,11,17

2.

theorem 1. *For all* $n \in \mathbb{N}$,

$$0^2 + 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Base Case: $0^2 = \frac{0(0+1)(2(0)+1)}{6}$.
Inductive Step: Let $n \in \mathbb{N}$ be arbitrary. Suppose

$$0^2 + 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Then,

$$0^2 + 1^2 + 2^2 + \cdots + n^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$0^2 + 1^2 + 2^2 + \cdots + n^2 + (n+1)^2 = \frac{n^3 + 3n^2 + n + 6n^2 + 12n + 6}{6}$$

$$0^2 + 1^2 + 2^2 + \cdots + n^2 + (n+1)^2 = \frac{n^3 + 9n^2 + 13n + 6}{6}$$

$$0^2 + 1^2 + 2^2 + \cdots + n^2 + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$

11.

theorem 2. *For all integers* $n \in \mathbb{N}$, $9|(4^n + 6n - 1)$.

Proof. Base Case: $9(0) = 4^0 + 6(0) - 1 = 1 + 0 - 1$.
Inductive Step: Let $n \in \mathbb{N}$ be arbitrary. Suppose $9|(4^n + 6n - 1)$. Then, we can find $k \in \mathbb{Z}$, so that

$$9k = (4^n + 6n - 1).$$

Then,

$$9(4k - 2n + 1) = 4(9k) - 2n + 1$$

$$9(4k - 2n + 1) = 4(4^n + 6n - 1) - 18n + 9$$

$$9(4k - 2n + 1) = (4^{n+1} + 6n + 5)$$

$$9(4k - 2n + 1) = (4^{n+1} + 6(n+1) - 1)$$

□

17.
a. The failed to prove the base case - among other problems, including that the theorem asserted is not correct.
b.

theorem 3. *For all $n \in \mathbb{N}$,*

$$1(3^0) + 3(3^1) + \cdots (2n+1)3^n = n3^{n+1} + 1$$

Proof. Base Case: $1 = 1(3^0) = 0(3^{0+1}) + 1$.
Inductive Step: Let $n \in \mathbb{N}$ be arbitrary. Suppose

$$1(3^0) + 3(3^1) + \cdots (2n+1)3^n = n3^{n+1} + 1$$

Then,

$$1(3^0) + 3(3^1) + \cdots (2n+1)3^n + (2n+3)3^{n+1} = n3^{n+1} + 1 + (2n+3)3^{n+1}$$

$$1(3^0) + 3(3^1) + \cdots (2n+1)3^n + (2n+3)3^{n+1} = (3n+3)3^{n+1}$$
$$1(3^0) + 3(3^1) + \cdots (2n+1)3^n + (2n+3)3^{n+1} = (n+1)3^{n+2}$$

□