HW §4.1 Numbers 2,5,7,9

2.

a. $\{(x, y) \in P \times C | x \text{ lives in } y\}$ For example (Dr. Iskra, Emory)

b. $\{(x, y) \in C \times \mathbb{N} | x \text{ has } y \text{ people}\}$

5.

theorem 1. *Suppose A, B and C are sets. Then,*

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

Proof. Let p be arbitrary. Suppose $p \in A \times (B \cup C)$. Then, $p = (x, y)$ where $x \in A$ and $y \in B \cup C$.
Suppose $y \in B$. Then $p = (x, y) \in A \times B$. Thus $p \in (A \times B) \cup (A \times C)$
Suppose $y \in C$. Then $p = (x, y) \in A \times C$. Thus $p \in (A \times B) \cup (A \times C)$
Thus, in either of these two exhaustive cases, if $p \in A \times (B \cup C)$, then $p \in (A \times B) \cup (A \times C)$. Since p was arbitrary,

$$A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$$

Now suppose $p \in (A \times B) \cup (A \times C)$. Then, either $p \in A \times B$ or $p \in A \times C$.
Suppose $p \in A \times B$. Then, $p = (x, y)$ where $x \in A$ and $y \in B$. Thus, $y \in B \cup C$. Thus, $p = (x, y) \in A \times (B \cup C)$.
Suppose $p \in A \times C$. Then, $p = (x, y)$ where $x \in A$ and $y \in C$. Thus, $y \in B \cup C$. Thus, $p = (x, y) \in A \times (B \cup C)$.
Thus, in either of these two exhaustive cases, if $p \in (A \times B) \cup (A \times C)$, then $p \in A \times (B \cup C)$. Since p was arbitrary,

$$A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$$

□

theorem 2. *Suppose A, B, C and C are sets. Then,*

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

Proof. Let $p \in (A \times B) \cap (C \times D)$. Then, $p \in (A \times B)$ and $p \in (C \times D)$. Thus, $p = (x, y)$ where $x \in A$ and $y \in B$ and $x \in C$ and $y \in D$. Thus, $x \in A \cap C$ and $y \in B \cap D$. Thus, $p = (x, y) \in (A \cap C) \times (B \cap D)$. Since p was arbitrary,

$$(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D)$$

Let $p \in (A \cap C) \times (B \cap D)$. Then, $p = (x, y)$ where $x \in (A \cap C)$ and $y \in (B \cap D)$. Thus, $x \in A$ and $y \in B$ and $x \in C$ and $y \in D$. Thus, $x \in A \times B$ and $y \in C \times D$. Thus, $p = (x, y) \in (A \times B) \cap (C \times D)$. Since p was arbitrary,

$$(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D)$$

Since

$$(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D)$$

and,

$$(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D)$$

we have

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

$\square$

7.mn

9.

theorem 3. *For any sets A, B, C and D*

$$(A \times B) \setminus (C \times D) = [A \times (B \setminus D)] \cup [(A \setminus C) \times B]$$

Proof. Let p be arbitrary. Suppose $p \in (A \times B) \setminus (C \times D)$. Then, $p \in A \times B$ and $p \notin C \times D$. Thus, $p = (x, y)$ where $x \in A$ and $y \in B$ and either $x \notin C$ or $y \notin D$.
Suppose $y \notin D$. Then, $y \in B \setminus D$. Thus, $p = (x, y) \in A \times (B \setminus D)$, thus $p \in [A \times (B \setminus D)] \cup [(A \setminus C) \times B]$. Since p was arbitrary,

$$(A \times B) \setminus (C \times D) \subseteq [A \times (B \setminus D)] \cup [(A \setminus C) \times B]$$

Let p be arbitrary and suppose $p \in [A \times (B \setminus D)] \cup [(A \setminus C) \times B]$ Then, $p \in A \times (B \setminus D)$ or $p \in (A \setminus C) \times B$.
Suppose $p \in A \times (B \setminus D)$. Then, $p = (x, y)$ where $x \in A$ and $y \in B \setminus D$. Then, $y \in B$ and $y \notin D$. Thus, $p = (x, y) \in A \times B$ and $(x, y) \notin C \times D$. Thus, $p \in (A \times B) \setminus (C \times D)$. Since p was arbitrary,

$$[A \times (B \setminus D)] \cup [(A \setminus C) \times B] \subseteq (A \times B) \setminus (C \times D)$$

Suppose $p \in A \setminus C \times B$. Then, $p = (x, y)$ where $x \in A \setminus C$ and $y \in B$. Then, $x \in A$ and $x \notin C$. Thus, $p = (x, y) \in A \times B$ and $(x, y) \notin C \times D$. Thus, $p \in (A \times B) \setminus (C \times D)$. Since p was arbitrary,

$$[A \times (B \setminus D)] \cup [(A \setminus C) \times B] \subseteq (A \times B) \setminus (C \times D)$$

Since

$$(A \times B) \setminus (C \times D) \subseteq [A \times (B \setminus D)] \cup [(A \setminus C) \times B]$$

and

$$[A \times (B \setminus D)] \cup [(A \setminus C) \times B] \subseteq (A \times B) \setminus (C \times D)$$

we have

$$(A \times B) \setminus (C \times D) = [A \times (B \setminus D)] \cup [(A \setminus C) \times B]$$

$\square$