HW §3.7 Numbers 2,3,5,8

2.

theorem 1. *Suppose A and B are sets. Then, $\mathcal{P}(A \setminus B) \setminus (\mathcal{P}(A) \setminus \mathcal{P}(B)) = \{\emptyset\}$.*

Proof. Suppose A and B are sets. Then, $\emptyset \subseteq A \setminus B$. Thus, $\emptyset \in \mathcal{P}(A \setminus B)$. Since $\emptyset \subseteq A$ and $\emptyset \subseteq B$, $\emptyset \in \mathcal{P}(A)$ and $\emptyset \in \mathcal{P}(B)$. Thus, $\emptyset \notin \mathcal{P}(A) \setminus \mathcal{P}(B)$. Thus, $\emptyset \in \mathcal{P}(A \setminus B) \setminus (\mathcal{P}(A) \setminus \mathcal{P}(B))$. □

3.

theorem 2. *Suppose A, B and C are sets. Then, the following are equivalent:*

1. $(A \triangle C) \cap (B \triangle C) = \emptyset$
2. $A \cap B \subseteq C \subseteq A \cup B$
3. $A \triangle C \subseteq A \triangle B$

Proof. 1⇒2: Suppose $(A \triangle C) \cap (B \triangle C) = \emptyset$. Let x be arbitrary. Suppose $x \in A \cap B$. Suppose $x \notin C$. Then, $x \in A \triangle C$, since $x \in A \setminus C$. Similarly, $x \in B \triangle C$. Thus, $x \in (A \triangle C) \cap (B \triangle C) = \emptyset$ which is a contradiction. Thus, $x \in C$. Since x was arbitrary, $A \cap B \subseteq C$.
Again, suppose $(A \triangle C) \cap (B \triangle C) = \emptyset$. Let x be arbitrary. Suppose $x \in C$. Suppose $x \notin A \cup B$. Then $x \notin A$, thus $x \in A \triangle C$. Also, $x \notin B$, so $x \in B \triangle C$. Thus, $x \in (A \triangle C) \cap (B \triangle C) = \emptyset$, a contradiction. Thus, $x \in A \cup B$. Since x was arbitrary, $C \subseteq A \cup B$.

2⇒3: Suppose $A \cap B \subseteq C \subseteq A \cup B$. Let x be arbitrary. Suppose $x \in A \triangle C$. Suppose $x \notin A \triangle B$. We then have two possibilities:
Case 1: $x \notin A \cup B$. Then, since $x \in A \triangle C$, $x \in C$. But this contradicts $C \subseteq A \cup B$. Thus, $x \in A \cup B$.
Case 2: $x \in A \cap B$. Then $x \in C$, thus $x \in A \cap C$. But $x \in A \triangle C$, so $x \notin A \cap C$ which is a contradiction. Thus, $x \notin A \cap B$. Thus, in either of the two possible cases implied by supposing $x \notin A \triangle B$, we arrive at a contradiction. Thus, $x \in A \triangle B$. Since x was arbitrary, $A \triangle C \subseteq A \triangle B$.

3⇒1: Suppose $A \triangle C \subseteq A \triangle B$. Let x be arbitrary. Suppose $x \in A \triangle C$. Then, $x \in A \triangle B$. Suppose $x \in A$. Then, $x \notin B$ and $x \notin C$. Thus, $x \notin B \cup C$, so $x \notin B \triangle C$. Suppose $x \notin A$. Then, $x \in B$ and $x \in C$. Thus, $x \in B \cap C$. Thus, $x \in B \triangle C$. Since x was arbitrary,$(A \triangle C) \cap (B \triangle C) = \emptyset$. □

5. There is a typo in the book in this problem. So, I've eliminated it as a required problem.

8.

theorem 3. *Suppose*

$$\lim_{x \to c} f(x) = L.$$

Then,

$$\lim_{x \to c} 7f(x) = 7L.$$

Proof. Suppose

$$\lim_{x \to c} f(x) = L.$$

Let $\epsilon > 0$ be arbitrary. Then we can find δ so that if

$$|x - c| < \delta$$

then

$$|f(x) - L| < \epsilon/7.$$

Thus, so that

$$|7f(x) - 7L| < \epsilon$$

Since ϵ was arbitrary we have that

$$\lim_{x \to c} 7f(x) = 7L.$$

□