HW §3.5 Numbers 2,4,5,10,17,27

2.

theorem 1. *Suppose A, B and C are sets. Then, $(A \cup B) \setminus C \subseteq A \cup (B \setminus C)$.*

Proof. Suppose A, B and C are sets. Let x be arbitrary. Suppose $x \in (A \cup B) \setminus C$. Then, $x \in A \cup B$ and $x \notin C$.
Case 1: Suppose $x \in A$. Then, $x \in A \cup (B \setminus C)$.
Case 2: Suppose $x \in B$. Then, since $x \notin C$, $x \in B \setminus C$. Thus, $x \in A \cup (B \setminus C)$.
Thus, since in either case, $x \in A \cup (B \setminus C)$, and, since x was arbitrary, $(A \cup B) \setminus C \subseteq A \cup (B \setminus C)$. □

4.

theorem 2. *Suppose $A \cap C \subseteq B \cap C$ and $A \cup C \subseteq B \cup C$. Then, $A \subseteq B$.*

Proof. Suppose $A \cap C \subseteq B \cap C$ and $A \cup C \subseteq B \cup C$. Let x be arbitrary. Suppose $x \in A$. Then, $x \in A \cup C$. Since $A \cup C \subseteq B \cup C$, $x \in B \cup C$. We consider two cases:
Case 1: Suppose $x \in B$. Then we are done.
Case 2: Suppose $x \in C$. Then, $x \in A \cap C$. Since $A \cap C \subseteq B \cap C$, $x \in B \cap C$. Thus, $x \in B$.
Since, in either case, if $x \in A$, then $x \in B$. Since x was arbitrary, $A \subseteq B$. □

5.

theorem 3. *Let A and B be two sets. If $A \triangle B \subseteq A$, then $B \subseteq A$.*

Proof. Let A and B be two sets. Suppose $A \triangle B \subseteq B$. Let x be arbitrary. Suppose $x \in B$. We consider two cases:
Case 1: Suppose $x \in A$. Then, we are done.
Case 2: Suppose $x \notin A$. Then, $x \in B \setminus A$. Also, $x \notin A \setminus B$. Thus, $x \in A \triangle B$. Since $A \triangle B \subseteq B$, $x \in A$, which is a contradiction. Thus, $x \in A$. Since x was arbitrary, $B \subseteq A$ □

10.

theorem 4. *For every real number x, if $|x - 3| > 3$, then $x^2 > 6x$.*

Proof. Let $x \in \mathbb{R}$ be arbitrary. Suppose $|x - 3| > 3$. We consider two cases:
Case 1: Suppose $x - 3 > 0$. Then, $|x - 3| = x - 3$. Thus, $x - 3 > 3$, so $x > 6$. Since $x > 0$, $x^2 > 6x$.
Case 2: Suppose $x - 3 < 0$. Then, $|x - 3| = 3 - x$. Thus, $3 - x > 3$, so $x < 0$. Since $x < 0$, $x^2 > 0 > 6x$.
Thus, in either case, $x^2 > 6x$. □

17.

theorem 5. *Suppose $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ are families of sets and for every $A \in \mathcal{F}$ and every $B \in \mathcal{G}$, $A \cup B \in \mathcal{H}$. Then $\cap\mathcal{H} \subseteq (\cap\mathcal{F}) \cup (\cap\mathcal{G})$.*

Proof. Suppose $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ are families of sets and for every $A \in \mathcal{F}$ and every $B \in \mathcal{G}$, $A \cup B \in \mathcal{H}$. Let x be arbitrary. Suppose $x \in \cap\mathcal{H}$. Suppose $x \notin (\cap\mathcal{F})$. Then, we can find $A \in \mathcal{F}$ so that $x \notin A$. Let $B \in \mathcal{G}$ be arbitrary. Then, $A \cup B \in \mathcal{H}$. Since $x \in \cup\mathcal{H}$, $x \in A \cup B$. Since $x \notin A$, $x \in B$. Since B was arbitrary, $x \in \cup\mathcal{G}$. Thus, $x \in (\cap\mathcal{F}) \cup (\cap\mathcal{G})$. Since x was arbitrary, $\cap\mathcal{H} \subseteq (\cap\mathcal{F}) \cup (\cap\mathcal{G})$. □

27. The proof is correct.