HW §3.4 Numbers 2,4,5,8,18,20

2.

theorem 1. *Suppose $A \subseteq B$ and $A \subseteq C$. Then $A \subseteq B \cap C$.*

Proof. Suppose $A \subseteq B$ and $A \subseteq C$. Let x be arbitrary. Suppose $x \in A$. Since $A \subseteq B$, $x \in B$. Since $A \subseteq C$, $x \in C$.Since $x \in B$ and $x \in C$, $x \in B \cap C$. Since x was arbitrary, $A \subseteq B \cap C$. □

4.

theorem 2. *If $A \subseteq B$ and $A \nsubseteq C$, then $B \nsubseteq C$.*

Proof. Suppose $A \subseteq B$ and $A \nsubseteq C$. Then, we can find x_0 so that $x_0 \in A$ and $x_0 \notin C$. Since $x_0 \in A$ and $A \subseteq B$, $x_0 \in B$. Since $x_0 \in B$ and $x_0 \notin C$, $B \nsubseteq C$. □

5.

theorem 3. *If $A \subseteq B \setminus C$ and $A \neq \emptyset$, then $B \nsubseteq C$*

Proof. Suppose $A \subseteq B \setminus C$ and $A \neq \emptyset$. Since $A \neq \emptyset$, we can find $x_0 \in A$. Since $A \subseteq B \setminus C$, $x_0 \in B \setminus C$. Thus, $x_0 \in B$ and $x_0 \notin C$. Thus, $B \nsubseteq C$. □

18.

theorem 4. *Suppose $\mathcal{F}$ and $\mathcal{G}$ are families of sets. Then $(\cup\mathcal{F}) \cap (\cup\mathcal{G}) \subseteq \cup(\mathcal{F} \cap \mathcal{G})$ iff $\forall A \in \mathcal{F} \forall B \in \mathcal{G}(A \cap B \subseteq \cup(\mathcal{F} \cap \mathcal{G})$.*

Proof. Suppose $(\cup\mathcal{F}) \cap (\cup\mathcal{G}) \subseteq \cup(\mathcal{F} \cap \mathcal{G})$. Let $A \in \mathcal{F}$ be arbitrary. Let $B \in \mathcal{G}$ be arbitrary. Let x be arbitrary. Suppose $x \in A \cap B$. Thus, $x \in A$, so $x \in \cup\mathcal{F}$. Thus, $x \in B$, so $x \in \cup\mathcal{G}$. Thus, $x \in (\cup\mathcal{F}) \cap (\cup\mathcal{G})$so, because $(\cup\mathcal{F}) \cap (\cup\mathcal{G}) \subseteq \cup(\mathcal{F} \cap \mathcal{G})$, $x \in \cup(\mathcal{F} \cap \mathcal{G})$. Since x was arbitrary $(A \cap B \subseteq \cup(\mathcal{F} \cap \mathcal{G})$. Since A and B was arbitrary $\forall A \in \mathcal{F} \forall B \in \mathcal{G}(A \cap B \subseteq \cup(\mathcal{F} \cap \mathcal{G})$.
Suppose that $\forall A \in \mathcal{F} \forall B \in \mathcal{G}(A \cap B \subseteq \cup(\mathcal{F} \cap \mathcal{G})$. Let x be arbitrary. Suppose $x \in (\cup\mathcal{F}) \cap (\cup\mathcal{G})$. Then we can find $A \in \mathcal{F}$ so that $x \in A$. We can also find $B \in \mathcal{G}$ so that $x \in B$. Thus, $x \in A \cap B$. Since $\forall A \in \mathcal{F} \forall B \in \mathcal{G}(A \cap B \subseteq \cup(\mathcal{F} \cap \mathcal{G})$, $x \in \cup(\mathcal{F} \cap \mathcal{G})$. Since x was arbitrary, $(\cup\mathcal{F}) \cap (\cup\mathcal{G}) \subseteq \cup(\mathcal{F} \cap \mathcal{G})$. □

20.

a.

theorem 5. *Suppose $\mathcal{F}$ and $\mathcal{G}$ are sets. Then, $(\cup\mathcal{F}) \setminus (\cup\mathcal{G}) \subseteq \cup(\mathcal{F} \setminus \mathcal{G})$.*

Proof. Suppose $\mathcal{F}$ and $\mathcal{G}$ are sets. Let x be arbitrary. Suppose $x \in (\cup\mathcal{F}) \setminus (\cup\mathcal{G})$. Then we can find $A \in \mathcal{F}$ with $x \in A$. Suppose $A \in \mathcal{G}$. Then, $x \in \cup\mathcal{G}$, which it is not. Thus, $A \notin \mathcal{G}$. Thus, $A \in \mathcal{F} \setminus \mathcal{G}$, thus $x \in \cup(\mathcal{F} \setminus \mathcal{G})$. Since x was arbitrary, $(\cup\mathcal{F}) \setminus (\cup\mathcal{G}) \subseteq \cup(\mathcal{F} \setminus \mathcal{G})$. □

b. The proof argues that because there *is* an $A \in \mathcal{G}$ with $x \notin A$, it must follow that $x \notin \cup\mathcal{G}$. However, in order to show that $x \notin \cup\mathcal{G}$ we must show that *for all* $A \in \mathcal{G}$, $x \notin A$.
c.

theorem 6. $\cup(\mathcal{F} \setminus \mathcal{G}) \subseteq (\cup\mathcal{F}) \setminus (\cup\mathcal{G})$ *iff* $\forall A \in (\mathcal{F} \setminus \mathcal{G})\forall B \in \mathcal{G}(A \cap B \neq \emptyset)$.

Proof. Suppose $\cup(\mathcal{F}\setminus\mathcal{G}) \subseteq (\cup\mathcal{F})\setminus(\cup\mathcal{G})$. Let $A \in (\mathcal{F}\setminus\mathcal{G})$ be arbitrary. Let $B \in \mathcal{G}$ be arbitrary. Suppose $x \in A$. Then, $x \in \cup(\mathcal{F}\setminus\mathcal{G})$. Since $\cup(\mathcal{F}\setminus\mathcal{G}) \subseteq (\cup\mathcal{F})\setminus(\cup\mathcal{G})$, $x \in (\cup\mathcal{F}) \setminus (\cup\mathcal{G})$. Since $B \in \mathcal{G}$, $x \notin B$. Thus, since x was arbitrary, $A \cap B = \emptyset$. Suppose $\forall A \in (\mathcal{F} \setminus \mathcal{G})\forall B \in \mathcal{G}(A \cap B \neq \emptyset)$. Let x be arbitrary. Suppose $x \in \cup(\mathcal{F}\setminus\mathcal{G})$. Then we can find $A \in \mathcal{F}\setminus\mathcal{G})$ so that $x \in A$. Thus, $x \in \cup\mathcal{F}$. Let $B \in \mathcal{G}$ be arbitrary. Suppose $x \in B$. Then, $x \notin B$ since $A \cap B = \emptyset$. Thus, $x \notin \cup\mathcal{G}$. Thus, $x \in (\cup\mathcal{F}) \setminus (\cup\mathcal{G})$. Since x was arbitrary, $\cup(\mathcal{F} \setminus \mathcal{G}) \subseteq (\cup\mathcal{F}) \setminus (\cup\mathcal{G})$. □

d. Let $A = \{a\}$ and $B := \{a, b\}$. Let $\mathcal{F} = \{A, B\}$ and $\mathcal{G} = \{A\}$. Then, $\cup(\mathcal{F} \setminus \mathcal{G}) = \{a, b\}$. On the other hand, $(\cup\mathcal{F}) \setminus (\cup\mathcal{G}) = \{a\}$.