

HW §3.3 Numbers 2,4,13,16,21,24

2.

**theorem 1.** *Suppose  $A$  and  $B \setminus C$  are disjoint. Then,  $A \cap B \subseteq C$ .*

*Proof.* Suppose  $A$  and  $B \setminus C$  are disjoint. Let  $x$  be arbitrary. Suppose  $x \in A \cap B$ . Then, since  $x \in A$ ,  $x \notin B \setminus C$ . Thus, either  $x \notin B$  or  $x \in C$ . But  $x \in B$ , so  $x \in C$ . Since  $x$  was arbitrary,  $A \cap B \subseteq C$ .  $\square$

4.

**theorem 2.** *Suppose  $A \subseteq \mathcal{P}(A)$ . Then,  $\mathcal{P}(A) \subseteq \mathcal{P}(\mathcal{P}(A))$ .*

*Proof.* Suppose  $A \subseteq \mathcal{P}(A)$ . Let  $x$  be arbitrary. Suppose  $x \in \mathcal{P}(A)$ . Then,  $x \subseteq A$ . Since  $A \subseteq \mathcal{P}(A)$ ,  $x \in \mathcal{P}(\mathcal{P}(A))$ . Since  $x$  was arbitrary,  $\mathcal{P}(A) \subseteq \mathcal{P}(\mathcal{P}(A))$ .  $\square$

13.

**theorem 3.** *Suppose  $\mathcal{F}$  and  $\mathcal{G}$  are families of sets. If  $\mathcal{F} \subseteq \mathcal{G}$ , then  $\cap \mathcal{G} \subseteq \cap \mathcal{F}$*

*Proof.* Suppose  $\mathcal{F}$  and  $\mathcal{G}$  are families of sets. Suppose  $\mathcal{F} \subseteq \mathcal{G}$ . Let  $x$  be arbitrary. Suppose  $x \in \cap \mathcal{G}$ . Let  $A$  be arbitrary and suppose  $A \in \mathcal{F}$ . Then, since  $\mathcal{F} \subseteq \mathcal{G}$ ,  $A \in \mathcal{G}$ . Thus,  $x \in A$ . Since  $A$  was arbitrary,  $x \in \cap \mathcal{F}$ . Since  $x$  was arbitrary,  $\cap \mathcal{G} \subseteq \cap \mathcal{F}$ .  $\square$

16.

**theorem 4.** *Suppose  $B$  is a set and  $\mathcal{F}$  a family of sets. If  $\mathcal{F} \subseteq \mathcal{P}(B)$ , then  $\cup \mathcal{F} \subseteq B$ .*

*Proof.* Suppose  $B$  is a set and  $\mathcal{F}$  a family of sets. Suppose  $\mathcal{F} \subseteq \mathcal{P}(B)$ . Let  $x$  be arbitrary. Suppose  $x \in \cup \mathcal{F}$ . Then, we can find  $A \in \mathcal{F}$  with  $x \in A$ . Since  $A \in \mathcal{F}$  and  $\mathcal{F} \subseteq \mathcal{P}(B)$ ,  $A \in \mathcal{P}(B)$ . Thus,  $A \subseteq B$ . Since  $x \in A$ , and  $A \subseteq B$ ,  $x \in B$ . Since  $x$  was arbitrary,  $\cup \mathcal{F} \subseteq B$ .  $\square$

21.

a. The proof needs to start off by assuming that  $x$  is an arbitrary element of  $B$ , not  $A$ .

b. Let  $A := \{0\}$  and  $B := \mathbb{R}$ .

24.

a. The proof does not assume that  $x$  and  $y$  are arbitrary, since it insists that they are equal.

b. The theorem is incorrect. Let  $x = 1$  and  $y = 0$ , for example (there are many, many other counter-examples)