HW §3.2 Numbers 3,4,7,10,12

3.

theorem 1. *Suppose $A \subseteq C$ and B and C are disjoint. Prove that if $x \in A$, then $x \notin B$.*

Proof. Suppose $A \subseteq C$ and B and C are disjoint. Suppose $x \in A$. Suppose $x \in B$. Then, since $A \subseteq C$, $x \in C$. But then, $x \in B \cap C$, which is a contradiction. Thus, $x \notin B$. □

4.

theorem 2. *Suppose that $A \setminus B$ is disjoint from C and $x \in A$. Prove that if $x \in C$, then $x \in B$.*

Proof. Suppose that $A \setminus B$ is disjoint from C and $x \in A$. Suppose that $x \in C$. Then, $x \notin A \setminus B$. Since $x \in A$, $x \in B$. □

7.

theorem 3. *Suppose that $y + x = 2y - x$ and x and y are not both zero. Prove that $y \neq 0$.*

Proof. Suppose that $y + x = 2y - x$ and x and y are not both zero. Suppose that $y = 0$. Then $x \neq 0$. Thus, $x = -x$, and so $x = 0$. But this is a contradiction. Thus, $y \neq 0$. □

10.

theorem 4. *Suppose that x and y are real numbers. Prove that if $x \neq 0$, then if $y = \frac{3x^2+2y}{x^2+2}$ then $y = 3$.*

Proof. Suppose that x and y are real numbers. Suppose that $x \neq 0$. Suppose that

$$y = \frac{3x^2 + 2y}{x^2 + 2}$$

Then,

$$yx^2 + 2y = 3x^2 + 2y$$
$$yx^2 = 3x^2$$
$$yx^2 - 3x^2 = 0$$
$$x^2(y - 3) = 0$$

Since $x \neq 0$, $y = 3$. □

12

a. It is not true that because $B \subseteq C$ $x \notin B$ implies that $x \notin C$.

b. Let A be the set of students in our 201 class, B be the set of students at UVA and C the set of all college students in Virginia. Then, any one of you is A but not in B.