HW §3.1 Numbers 2,3,5,8,15,16

2.
a. Hypotheses:

1. $b^2 > 4ac$.

Conclusions:

1. The quadratic equation $ax^2 + bx + c = 0$ has exactly two solutions.

b. It must be true for arbitrary x. *What* a , b, and c are, however, depend on x.
c. Since $(-5)^2 > 4(2)(3)$, the theorem implies that the equation $ax^2+bx+c = 0$ has exactly two solutions. In this case the solutions are $x = -1/2$ and $x = 3$.
d. Nothing.
3.
Hypotheses:

1. $n \in \mathbb{N}$.
2. $n > 2$.
3. n is not prime.

Conclusions:

1. $2n + 13$ is not prime.

When $n = 8$ (which is not prime), $2n + 13 = 29$, which is.

5.

theorem 1. *Suppose a and b are real numbers. Prove that if $a < b < 0$, then $a^2 > b^2$*

Proof. Suppose a and b are real numbers. Suppose $a < b < 0$. Then, multiplying both sides of the inequality $a < b$ by a gives $a^2 > ab$. Mulitplying both sides by b gives $ab > b^2$. Thus, $a^2 > ab > b^2$, that is, $a^2 > b^2$. □

8.

theorem 2. *Suppose $A \setminus B \subseteq C \cap D$ and $x \in A$. Prove that if $x \notin D$, then $x \in B$.*

Proof. Suppose $A \setminus B \subseteq C \cap D$ and $x \in A$. Suppose that $x \notin D$. Then, $x \notin C \cap D$. Thus, $x \notin A \setminus B$. Since $x \in A$, $x \in B$. □

15.

a. It assumes that which is to be proven.

b.

theorem 3. *Suppose* $x \in \mathbb{R}$ *and* $x \neq 4$. *If* $\frac{2x-5}{x-4} = 3$, *then* $x = 7$.

Proof. Suppose $x \in \mathbb{R}$ and $x \neq 4$. Suppose $\frac{2x-5}{x-4} = 3$. Then, $2x - 5 = 3x - 12$, or $x = 7$. □

16.

a. In the third sentence, it is claimed that "since $x \neq 3$, $x^2 \neq 9$. However, $(-3)^2 = 9$.

b. $x = -3$ and $y = 1$ is a counterexample.