HW §2.3 Numbers 3,6,7,14

3. $\mathcal{P}(\{\emptyset\}) = \{\emptyset, \{\emptyset\}$.

6. a.

$$A_2 = \{2,3,1,4\}$$

$$A_3 = \{3,4,2,6\}$$

$$A_4 = \{4,5,3,8\}$$

$$A_5 = \{5,6,4,10\}$$

b.

$$\cap_{i\in I} A_i = \{4\}$$

$$\cup_{i\in I} A_i = \{1,2,3,4,5,6,8,10\}$$

7.

$$\bigcup_{y\in Y} P = \{Bach, Goethe, Hume, Mozart, Washington\}$$

$$\bigcap_{y\in Y} P = \{Goethe, Hume, Washington\}$$

14.

a. If $\mathcal{F} = \emptyset$, then, because the statement $x \in \bigcup \mathcal{F}$ is equivalent to the statement $\exists A(A \in \mathcal{F} \wedge x \in\ A)$, and because the statement $A \in \mathcal{F}$ is always false (thus, the whole statement $\exists A(A \in \mathcal{F} \wedge x \in\ A)$ is always false) we see that $\bigcup \mathcal{F} = \emptyset$.
b. Let **U** be the universe of discourse. If $\mathcal{F} = \emptyset$, then, because the statement $x \in \bigcap \mathcal{F}$ is equivalent to the statement $\forall A(A \in \mathcal{F} \to x \in\ A)$, and because the statement $\exists A(A \in \mathcal{F} \wedge x \in\ A$ is always false we see that the whole implication $A \in \mathcal{F} \wedge x \in\ A)$ is always true, regardless of A or x. Thus, $\bigcap \mathcal{F} = \mathbf{U}$.