

### HW §2.2 Numbers 2,3,7,10

2.

- Everyone in the freshman class has a roommate.
- Either there is someone who doesn't like anybody, or there is someone who likes everyone.
- $\exists a \in A \forall b \in B [(a \in C \wedge b \notin C) \vee (a \notin C \wedge b \in C)]$
- $\exists y > 0 \forall x (ax^2 + bx + c \neq y)$ .

3.

- False. For example,  $x = 5$  makes the statement false.
- False. Both  $x = 7$  and  $x = 1$  make the statement true, so the solution is not unique.
- True. Only  $x = 9$  makes the statement true, since the universe is  $\mathbb{N}$ .
- True.  $x = y = 9$  make the statement true.

7.

$$\begin{aligned} & \exists x (P(x) \rightarrow Q(x)) \\ &= \exists x (\neg P(x) \vee Q(x)) \\ &= \exists x (\neg P(x)) \vee \exists x (Q(x)) \\ &= \neg \forall x (P(x)) \vee \exists x (Q(x)) \\ &= \forall x (P(x)) \rightarrow \exists x (Q(x)) \end{aligned}$$

10.

a.

$$\begin{aligned} & \exists x \in A P(x) \vee \exists x \in B P(x) \\ &= \exists x (x \in A \wedge P(x)) \vee \exists x (x \in B \wedge P(x)) \\ &= \exists x ((x \in A \wedge P(x)) \vee (x \in B \wedge P(x))) \\ &= \exists x ((x \in A \vee x \in B) \wedge P(x)) \\ &= \exists x \in A \cup B P(x) \end{aligned}$$

- This is not true. For example, let  $P(x) : x^2 = 9$ ,  $A := \mathbb{Z}^+$  and  $B := \mathbb{Z}^-$ , the positive and negative integers respectively.