Solutions: §6.4 4,14,16,22

4.

$$W = \int_1^2 \cos(\pi x/3)dx$$
$$= (\frac{3}{\pi}\sin(\pi x/3)|_1^2$$
$$= \frac{3}{\pi}(\frac{\sqrt{3}}{2} - \frac{1}{2})$$

The work done moving the particle from 1 to 2 is equal to the sum of the work done moving the particle from 1 to 1.5 and the work done moving the particle from 1.5 to 2.

14.

$$W = \int_0^{10} 88.4x dx$$
$$= 44.2x^2|_0^{10}$$
$$= 4420J$$

16. Divide the length the rope to be pulled up into n segments each of which has length Δx. For purposes of estimation, suppose that the work done moving the bucket from the bottom of the i^{th} segment to the top is constant. If we further denote by x_i the distance the rope is from the top during this segment (this is why we have only an estimate from this calculation). Then, the weight of the bucket and water in this i^{th} segment is

$$4 + [40 - .1(80 - x)]lbs$$

So, the work done moving it over the distance of Δx is

$$4 + [40 - .1(80 - x)]\Delta x ftlbs$$

So, the work done is approximated by adding the work done over each of these n subintervals.

$$W \sum_{i=1}^{n} 4 + [40 - .1(80 - x)]\Delta x ftlbs$$

Improving the estimate by shrinking Δx we get

$$W \lim_{n\to\infty} \sum_{i=1}^{n} 4 + [40 - .1(80 - x)]\Delta x ftlbs$$

So,

$$W = \int_0^{80} 4 + [40 - .1(80 - x)]dx ftlbs$$

$$
\begin{aligned}
W &= \int_0^{80} (36 + .1x) dx ftlbs \\
&= 36x + .05x^2|_0^{80} \\
&= 3200
\end{aligned}
$$

22. This turned out to be trickier to integrate than I had anticipated. In order to evaluate the integral you need to define the width of the i^{th} slab in terms of the angle formed by a radius meeting the edge of the circular face of the tank and the end the i^{th} slab. If we call this angle θ, then the width of any slab is $1.5 \sin \theta$. In terms of θ the distance from the top of any slab in the top half of the tank $0 \le \theta \le \pi/2$ is $2.5 - 1.5 \cos \theta$ and in the bottom half of the tank is $2.5 + 1.5 \cos \theta$. Thus, we have the following calculation for the work done:

$$
W = 9,800[\int_0^{\pi/2} (2.5 - 1.5 \cos \theta)(1.5 \sin \theta) + \int_{\pi/2}^{\pi} (2.5 + 1.5 \cos \theta)(1.5 \sin \theta)]
$$

$$
\begin{aligned}
W = 9,800[-4.5 \cos \theta|_0^{\pi/2} - 1.125 \sin^2 \theta|_0^{\pi/2} + -4.5 \cos \theta|_{\pi/2}^{\pi} + 1.125 \sin^2 \theta|_{\pi/2}^{\pi}] \\
= 9,800[(4.5 - 1.125) + (4.5 + 1.125)] \\
= 88,200J
\end{aligned}
$$