Solutions: §7.3. 6-20 even; 24-30 even

6. Let $x = 2\tan\theta$. Then, $dx = 2\sec^2\theta d\theta$, so

$$\int_0^2 x^3\sqrt{x^2+4}dx = \int_0^{\pi/4} \tan^3\theta\sqrt{4\tan^2\theta+4}d\theta$$

$$= \int_0^{\pi/4} \tan^3\theta 2\sec\theta d\theta$$

$$= \int_0^{\pi/4} (\sec^2\theta - 1)2\sec\theta\tan\theta d\theta$$

Let $u = \sec\theta$. Then, $du = \sec\theta\tan\theta d\theta$. So,

$$= \int_1^{\sqrt{2}} (u^2-1)du$$

$$= \frac{1}{3}u^3 - u|_1^{\sqrt{2}}$$

$$= \sqrt{2}^3 - \sqrt{2}$$

8. Let $x = a\sec\theta$. Then, $dx = a\sec\theta\tan\theta d\theta$, so

$$\int \frac{\sqrt{x^2-a^2}}{x^4}dx = \int \frac{\sqrt{a^2\sec^2\theta - a^2}}{\sec^4\theta}\sec\theta\tan\theta d\theta$$

$$= \int \frac{\tan^2\theta}{\sec^3\theta}d\theta$$

$$= \int \frac{\sin^2\theta}{\cos^2\theta}\cos^3\theta d\theta$$

$$= \int \sin^2\theta\cos\theta d\theta$$

$$= \frac{1}{3}\sin^3\theta + C$$

$$= \frac{\sqrt{(x^2-a^2)^3}}{3a^3}$$

10. Let $t = \sqrt{2}\tan\theta$, then $dt = \sqrt{2}\sec^2\theta$. So,

$$\int \frac{t^5}{\sqrt{t^2+2}}dt = \int \frac{\sqrt{2^5}\tan^5\theta}{\sqrt{2\tan^2\theta+2}}\sqrt{2}\sec^2\theta$$

$$= 4\sqrt{2}\int \frac{\tan^5\theta}{\sec\theta}\sec^2\theta d\theta$$

$$= 4\sqrt{2}\int \frac{\sin^5\theta}{\cos^4\theta}d\theta$$

$$= 4\sqrt{2}\int \frac{(1-\cos^2\theta)^2}{\cos^4\theta}\sin\theta d\theta$$

$$= 4\sqrt{2}\int (\sec^4\theta - 2\sec^2\theta + 1)\sin\theta d\theta$$

$$= 4\sqrt{2}\int \tan\theta\sec^3\theta - \tan\theta\sec\theta + \sin\theta d\theta$$

$$= 4\sqrt{2}(\frac{1}{3}\sec^3\theta - \sec\theta - \cos\theta) + C$$

$$= 4\sqrt{2}(\frac{\sqrt{(t^2+2)^3}}{3\sqrt{8}} - \sqrt{\frac{t^2+2}{2}} - \sqrt{\frac{2}{t^2+2}}$$

12. Let $x = 2\tan\theta$. Then, $dx = 2\sec^2\theta d\theta$. So,

$$\int_0^1 x\sqrt{x^2+4} = \int_0^{\pi/6} 2\tan\theta\sec^3\theta d\theta$$

$$= \int_0^{\pi/6} 2\tan\theta\sec^3\theta d\theta$$

$$= \frac{1}{3}\sec^3\theta|_0^{\pi/6}$$

$$= \frac{1}{3}(\frac{\sqrt{27}}{8} - 1)$$

14. Let $u = \sqrt{5}\sin\theta$. Then, $du = \sqrt{5}\cos\theta d\theta$. So,

$$= \int \frac{du}{u\sqrt{5-u^2}} = \frac{\sqrt{5}\cos\theta d\theta}{\sqrt{5}\sin\theta\sqrt{5-5\sin^2\theta}}$$

$$= \int \frac{\sqrt{5}\cos\theta d\theta}{5\sin\theta\cos\theta}$$

$$= \frac{\sqrt{5}}{5}\ln|\csc\theta - \cot\theta| + C$$

$$= \frac{\sqrt{5}}{5}\ln|\sqrt{\frac{5}{5-u^2}} - \frac{\sqrt{5-u^2}}{u}|$$

16. Let $x = 3/4\sec\theta$. Then, $dx = \frac{3}{4}\sec\theta\tan\theta$. So,

$$= \int \frac{dx}{x^2\sqrt{16x^2-9}} = \int \frac{\frac{3}{4}\sec\theta\tan\theta}{\frac{9}{16}\sec^2\theta\sqrt{9\sec^2\theta-9}}$$

$$= \int \frac{\frac{3}{4}\sec\theta\tan\theta}{\frac{27}{16}\sec^2\theta\tan\theta}$$

$$= \frac{1}{3}\int \cos\theta d\theta$$

$$= \frac{1}{3}\sin\theta + C$$

$$= \frac{\sqrt{16x^2 - 9}}{12x} + C$$

18. Let $x = \frac{b}{a}\sec\theta$. Then, $dx = \frac{b}{a}\sec\theta\tan\theta$. So,

$$\int \frac{dx}{[(ax)^2 - b^2]^{3/2}} = \int \frac{\frac{b}{a}\sec\theta\tan\theta}{[(a\frac{b}{a}\sec^2\theta) - b^2]^{3/2}}$$

$$= \int \frac{\frac{b}{a}\sec\theta\tan\theta d\theta}{[b\tan\theta]^3}$$

$$= \int \frac{\cos\theta d\theta}{ab^2\sin^2\theta}$$

Letting $u = \sin\theta$ and un-substituting, we get

$$= \frac{-1}{ab^2}\frac{\sqrt{(ax)^2 - b^2}}{4x} + C$$

20. Let $t = 5\sin\theta$. Then, $dt = 5\cos\theta d\theta$. So,

$$\int \frac{t}{\sqrt{25 - t^2}}dt = \int \frac{5\cos\theta d\theta}{\sqrt{25 - 25\sin^2\theta}}$$

$$= \int d\theta$$

$$= \theta$$

$$= \sin^{-1}(\frac{t}{5})$$

24.

$$\int \frac{dt}{\sqrt{t^2 - 6t + 13}} = \int \frac{dt}{\sqrt{(t-3)^2 + 4}}$$

Let $t - 3 = 2\tan\theta$. Then, $dt = 2\sec^2\theta d\theta$. Which gives

$$= \int \frac{2\sec^2\theta d\theta}{\sqrt{(4\tan^2 theta + 4}}$$

$$= \int \sec\theta d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln|\frac{\sqrt{t^2 - 6t + 13}}{2} + \frac{t-3}{2}| + C$$

26.

$$= \int \frac{x^2}{\sqrt{4x - x^2}}dx = \int \frac{x^2}{\sqrt{4 - (2-x)^2}}dx$$

Let $2 - x = 2\cos\theta$, then $dx = 2\sin\theta$. So,

$$= \int \frac{4\cos^2\theta}{\sqrt{4 - 4\cos^2\theta}} 2\sin\theta$$

$$= \int 2\cos^2\theta d\theta$$

$$= \theta + \frac{1}{2}\sin\theta + C$$

$$= \cos^{-1}(\frac{2-x}{2}) + \frac{1}{4}\sqrt{4x - x^2} + C$$

28.

$$= \int \frac{dx}{(5 - 4x - x^2)^{5/2}} = \frac{dx}{(1 - (2-x)^2)^{5/2}}$$

Let $2 - x = \cos\theta$. Then, $dx = \sin\theta$. So we get

$$= \int \frac{\sin\theta d\theta}{(1 - \cos^2\theta)^{5/2}}$$

$$= \int \csc^4\theta d\theta$$

$$= \int (1 + \cot\theta)\csc^2\theta d\theta$$

$$= -\cot\theta - \frac{1}{2}\cot^2\theta + C$$

$$= \frac{2-x}{\sqrt{-3 + 4x - x^2}} - \frac{1}{2}\frac{(2-x)^2}{-3 + 4x - x^2} + C$$