

**Solutions: §7.1. 4,10,16,28,34**

4.

Let  $u = x$  and  $dv = e^{-x}$ . Then,

$$\begin{aligned}\int x e^{-x} dx &= -x e^{-x} + \int e^{-x} dx \\ &= x e^{-x} - e^{-x} + C\end{aligned}$$

10. Let  $u = \sin^{-1} x$  and  $dv = dx$ . Then,

$$\int \sin^{-1} x dx = x \sin^{-1} x - \int \frac{x dx}{\sqrt{1-x^2}}$$

If we let  $w = 1 - x^2$ , we get

$$\begin{aligned}\int \sin^{-1} x dx &= x \sin^{-1} x - \int \frac{-1}{2} \frac{dw}{\sqrt{w}} \\ &= x \sin^{-1} x + \frac{1}{2} 2\sqrt{w} \\ &= x \sin^{-1} x + \sqrt{1-x^2}\end{aligned}$$

16. Let  $u = e^{-\theta}$  and  $dv = \cos 2\theta$ . Then,

$$\int e^{-\theta} \cos 2\theta d\theta = \frac{1}{2} e^{-\theta} \sin 2\theta - \int \frac{-1}{2} e^{-\theta} \sin 2\theta d\theta$$

Again, let  $u = e^{-\theta}$ , but now let  $dv = \sin 2\theta$ . Then,

$$\begin{aligned}\int e^{-\theta} \cos 2\theta d\theta &= \frac{1}{2} e^{-\theta} \sin 2\theta + \frac{1}{2} \left[ \frac{-1}{2} e^{-\theta} \cos 2\theta - \frac{1}{2} \int e^{-\theta} \cos 2\theta d\theta \right] \\ \int e^{-\theta} \cos 2\theta d\theta &= \frac{1}{2} e^{-\theta} \sin 2\theta + \frac{-1}{4} e^{-\theta} \cos 2\theta - \frac{1}{4} \int e^{-\theta} \cos 2\theta d\theta\end{aligned}$$

Thus,

$$\frac{5}{4} \int e^{-\theta} \cos 2\theta d\theta = \frac{1}{2} e^{-\theta} \sin 2\theta + \frac{-1}{4} e^{-\theta} \cos 2\theta$$

So,

$$\int e^{-\theta} \cos 2\theta d\theta = \frac{2}{5} e^{-\theta} \sin 2\theta + \frac{-1}{5} e^{-\theta} \cos 2\theta$$

28. We will first find an anti-derivative for the integrand:

Let  $u = \arctan(1/x)$  and  $dv = dx$ . Then,

$$\int \arctan(1/x) dx = x \arctan(1/x) - \int \frac{-x dx}{1+x^2}$$

Let  $w = 1 + x^2$

$$= x \arctan(1/x) - \int \frac{1-dw}{2w}$$

$$\begin{aligned}
&= x \arctan(1/x) - \frac{1}{2} \ln |w| \\
&= x \arctan(1/x) - \frac{1}{2} \ln |1 + x^2|
\end{aligned}$$

Thus,

$$\begin{aligned}
\int_1^{\sqrt{3}} \arctan(1/x) dx &= (x \arctan(1/x) - \frac{1}{2} \ln |1 + x^2|) \Big|_1^{\sqrt{3}} \\
&= (\sqrt{3} \frac{\pi}{6} - \frac{1}{2} \ln(4)) - (\frac{\pi}{4} + \frac{1}{2} \ln(2)) \\
&= (\sqrt{3} \frac{\pi}{6} - \frac{1}{2} \ln(2)) - \frac{\pi}{4}
\end{aligned}$$

34. Let  $z = \sqrt{x}$ . Then,  $dz = \frac{dx}{2\sqrt{x}}$ , or  $2\sqrt{x}dz = dx$ , or  $2zdz = dx$ . So,

$$\int_1^4 e^{\sqrt{x}} dx = \int_1^2 2ze^z dz$$

Let  $u = z$  and  $dv = e^z$ . Then,

$$\begin{aligned}
\int_1^2 2ze^z dz &= 2ze^z \Big|_1^2 - 2 \int_1^2 e^z dz \\
&= 4e^2 - 2e - 2e^z \Big|_1^2 \\
&= 4e^2 - 2e - 2e^2 + 2e \\
&= 2e^2
\end{aligned}$$