HW §7.5 Numbers 2,8,14,20,26,32,38,44,50,56,62,68,74,80

2.

$$\int \tan^3\theta d\theta = \int (1+\sec^2\theta)\tan\theta d\theta$$

$$= \ln|\sec\theta| + \frac{1}{2}\tan^2\theta + C$$

8.

$$\int_0^4 \frac{x-1}{x^2-4x-5}dx = \int_0^4 \frac{A}{x-5} + \frac{B}{x+1}dx$$

We have

$$x-1 = A(x+1) + B(x-5)$$

So, $A = 2/3$ and $B = 1/3$. So,

$$\int_0^4 \frac{x-1}{x^2-4x-5}dx = \int_0^4 \frac{2}{3(x-5)} + \frac{1}{3(x+1)}dx$$

$$= \frac{2}{3}\ln|x-5| + \frac{1}{3}\ln|x+1||_0^4$$

$$= \frac{-2}{3}\ln|5| + \frac{1}{3}\ln|5|$$

$$= \frac{-1}{3}\ln|5|$$

14. Let $w = \ln x$, then $dw = 1/x$. Then,

$$\int \frac{\sqrt{1+\ln x}}{x\ln x} = \int \frac{\sqrt{w+1}}{w}$$

Let $u = \sqrt{1+w}$, then $u^2 - 1 = w$ and $dw = 2udu$. Now we have

$$= \int \frac{2u^2}{u^2-1}du$$

$$= \int 2 + \frac{2}{u^2-1}du$$

$$= 2u - \ln|u+1| + \ln|u-1| + C$$

$$= 2\sqrt{1+w} - \ln|\sqrt{1+w}+1| + \ln|\sqrt{1+w}-1| + C$$

$$= 2\sqrt{1+\ln x} - \ln|\sqrt{1+\ln x}+1| + \ln|\sqrt{1+\ln x}-1| + C$$

20. Let $u = \sqrt[3]{x}$, then $du = \frac{1}{3\sqrt[3]{x^2}}dx$. Thus,$3u^2du = dx$. So,

$$\int e^{\sqrt[3]{x}}dx = \int 3u^2e^u du$$

Which, using integration by parts is

$$= u^2e^u - 2ue^u + 2e^u$$

$$= \sqrt[3]{x^2}e^{\sqrt[3]{x}} - 2\sqrt[3]{x}e^{\sqrt[3]{x}} + 2e^{\sqrt[3]{x}}$$

26. Let $u = x^3 - 2x - 8$, $du = 3x^2 - 2$. Then,

$$\int \frac{3x^2 - 2}{x^3 - 2x - 8}dx = \int \frac{du}{u}$$

$$= \ln|u| + C$$

$$= \ln|x^3 - 2x - 8| + C$$

32. Let $u = \sqrt{2x-1}$, then $\frac{u^2+1}{2} = x$ and $udu = dx$. So,

$$\int \frac{\sqrt{2x-1}}{2x+3}dx = \int \frac{u^2du}{u^2+4}$$

$$= \int 1 - \frac{4}{u^2+4}du$$

Letting $2\tan\theta = u$, we get

$$= u - \tan^{-1}(\frac{u}{2}) + C$$

$$= \sqrt{2x-1} - \tan^{-1}(\frac{\sqrt{2x-1}}{2}) + C$$

38.

$$\int_0^{\pi/4} \tan^5\theta \sec^3\theta d\theta = \int_0^{\pi/4} \tan^4\theta \sec^2\theta \tan\theta \sec\theta d\theta$$

$$= \int_0^{\pi/4} (\sec^2\theta - 1)^2 \sec^2\theta \tan\theta \sec\theta d\theta$$

Let $u = \sec\theta$ and $du = \sec\theta\tan\theta d\theta$. Then we have

$$= \int_1^{\sqrt{2}} (u^2-1)^2u^2du$$

$$= \frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3|_1^{\sqrt{2}}$$

$$= \frac{1}{7}\sqrt{2}^7 - \frac{2}{5}\sqrt{2}^5 + \frac{1}{3}\sqrt{2}^3 - \frac{1}{7} + \frac{2}{5} - \frac{1}{3}$$

44. Let $u = e^x$, then $\frac{du}{u} = dx$. So,

$$\int \sqrt{1+e^x}dx = \int \frac{\sqrt{1+u}}{u}du$$

Let $w = \sqrt{1+u}$, then $w^2 - 1 = u$ and $2wdw = du$ so we have

$$= \int \frac{2w^2}{w^2-1}dw$$

$$= \int 2 + \frac{2}{w^2-1} dw$$
$$= 2w + 2\ln|w-1| - 2\ln|w+1| + C$$
$$= 2\sqrt{1+u} + 2\ln|\sqrt{1+u}-1| - 2\ln|\sqrt{1+u}+1| + C$$
$$= 2\sqrt{1+e^x} + 2\ln|\sqrt{1+e^x}-1| - 2\ln|\sqrt{1+e^x}+1| + C$$

50. Let $u = \sqrt{4x+1}$, then $8udu = dx$ and $\frac{u^2-1}{4} = x$. Then,

$$\int \frac{1}{x^2\sqrt{4x+1}} dx = \frac{1}{16}\int \frac{8udu}{u(u^2-1)}$$
$$= \frac{1}{16}\int \frac{-4}{u+1} + \frac{4}{u-1} du$$
$$= \frac{1}{16}(-4\ln|u+1| + 4\ln|u-1|) + C$$

56. Using integration by parts and letting $u = \ln x$ and $dv = \frac{x}{\sqrt{x^2-1}}$ we get

$$\int \frac{x\ln x}{\sqrt{x^2-1}} dx = \ln x\sqrt{x^2-1} - \int \frac{\sqrt{x^2-1}dx}{x}$$

Let $x = \sec\theta$ and $dx = \sec\theta\tan\theta$ we have

$$= \ln x\sqrt{x^2-1} - \int \tan^2\theta d\theta$$
$$\ln x\sqrt{x^2-1} - \tan\theta - \theta + C$$
$$\ln x\sqrt{x^2-1} - \sqrt{1+x^2} - \sec^{-1} x + C$$

62. Let $u = x+1$, then $du = dx$ and $x = u-1$. So,

$$\int \frac{x^3}{(x+1)^{10}} dx = \int \frac{(u-1)^3}{u^{10}}$$
$$= \int u^{-7} - 3u^{-8} + 3u^{-9} - u^{-10} du$$
$$= \frac{-1}{6}(x+1)^{-6} + \frac{3}{7}(x+1)^{-7} - \frac{3}{8}(x+1)^{-8} + \frac{1}{9}(x+1)^{-9} + C$$

68.
$$\int \frac{1}{1+2e^x-e^{-x}} dx = \int \frac{e^x}{e^x+2e^{2x}-1} dx$$

Let $u = e^x$, then $du = e^x dx$, so we have

$$= \int \frac{du}{2u^2+u-1}$$
$$= \int \frac{\frac{-1}{3}}{2u-1} + \frac{\frac{-1}{3}}{u+1} du$$

$$= \frac{-1}{6}\ln|2u-1| + \frac{-1}{3}\ln|u+1| + C$$
$$= \frac{-1}{6}\ln|2e^x-1| + \frac{-1}{3}\ln|e^x+1| + C$$

74.

$$\int \frac{dx}{e^x - e^{-x}} = \int \frac{e^x dx}{e^{2x}-1}$$

Let $u = e^x$, then $du = e^x dx$. Then

$$= \int \frac{du}{u^2-1}$$
$$= \int \frac{1}{2(u-1)} + \frac{-1}{2(u+1)} du$$
$$= \frac{1}{2}\ln|u-1| + \frac{-1}{2}\ln|u+1| + C$$
$$= \frac{1}{2}\ln|e^x-1| + \frac{-1}{2}\ln|e^x+1| + C$$

80. Let $u = \sin^2 x$, then $du = 2\sin x \cos x dx$. So,

$$\int \frac{\sin x \cos x}{\sin^4 + \cos^4 x} dx = \int \frac{du}{2(u^2 + (1-u)^2)}$$
$$= \int \frac{du}{4u^2 - 2u + 1}$$
$$= \int \frac{du}{4(u - \frac{1}{4})^2}$$
$$= \frac{-1}{4u-1} + C$$
$$= \frac{-1}{4\sin^2 x - 1} + C$$