

HW §8.2 Numbers 4,6,8

4.

$$S = \int_1^2 2\pi \ln y \sqrt{1 + \frac{1}{y^2}} dy$$

6.

$$S = \int_2^6 2\pi \sqrt{9x - 18} \sqrt{1 + \frac{9}{4x - 8}} dx$$

$$S = 2\pi \int_2^6 \sqrt{9x + \frac{45}{2}} dx$$

Let $u = 9x + \frac{45}{2}$, then $1/9 du = dx$. Thus,

$$S = \frac{2}{9}\pi \int_{81/2}^{153/2} \sqrt{u} du$$

$$S = \frac{4}{27}\pi u^{3/2} \Big|_{81/2}^{153/2}$$

$$S = \frac{4}{27}\pi((153/2)^{3/2} - (81/2)^{3/2})$$

8.

$$S = \int_0^{\pi/6} 2\pi \cos 2x \sqrt{1 + 4 \sin^2 2x} dx$$

Let $u = 2 \sin 2x$. Then, $1/4 du = \cos 2x$. So,

$$S = \int_0^{\sqrt{3}} \frac{1}{2}\pi \sqrt{1 + u^2} du$$

Let $u = \tan \theta$. Then, $du = \sec^2 \theta$. So,

$$S = \int_0^{\pi/3} \frac{1}{2}\pi \sec^3 \theta$$

$$S = \frac{\pi}{4}(\sec \theta \tan \theta + \ln |\sec \theta + \tan \theta| \Big|_0^{\pi/3}$$

$$= \frac{\pi}{4}(2\sqrt{3} + \ln |2 + \sqrt{3}|$$