

HW §7.4 Numbers 2,4,14,18,20,24,28,38

4.

a.

$$\frac{x^3}{x^2 + 4x + 3} = x - 4 + \frac{13x + 12}{x^2 + 4x + 3} = x - 4 + \frac{A}{x + 3} + \frac{B}{x + 1}$$

b.

$$\frac{2x + 1}{(x + 1)^3(x^2 + 4)^2} = \frac{A}{x + 1} + \frac{B}{(x + 1)^2} + \frac{C}{(x + 1)^3} + \frac{Dx + E}{x^2 + 4} + \frac{Fx + G}{(x^2 + 4)^2}$$

14.

Set

$$\int \frac{1}{(x + a)(x + b)} dx = \int \frac{A}{x + a} Bx + b$$

Then,

$$1 = (A + B)x + (Ab + Ba)$$

So,  $A = \frac{1}{b-a}$  and  $b = \frac{1}{a-b}$ . Thus,

$$\begin{aligned} \int \frac{1}{(x + a)(x + b)} dx &= \int \frac{\frac{1}{b-a}}{x + a} \frac{1}{a-b} x + b \\ &= \frac{1}{b-a} \ln|x + a| + \frac{1}{a-b} \ln|x + b| \end{aligned}$$

18.

$$\int \frac{x^2 + 2x - 1}{x^3 - x} dx = \int \frac{A}{x} + \frac{B}{x - 1} + \frac{C}{x + 1} dx$$

So,

$$\frac{x^2 + 2x - 1}{x^3 - x} = \frac{A}{x} + \frac{B}{x - 1} + \frac{C}{x + 1}$$

Thus,

$$x^2 + 2x - 1 = A(x^2 - 1) + B(x^2 + x) + C(x^2 - x)$$

$$x^2 + 2x - 1 = (A + B + C)x^2 + (B - C)x - A$$

So,  $A = 1$ ,  $B = 1$  and  $C = -1$ . So,

$$\begin{aligned} \int \frac{x^2 + 2x - 1}{x^3 - x} dx &= \int \frac{1}{x} + \frac{1}{x - 1} + \frac{-1}{x + 1} dx \\ &= \ln|x| + \ln|x - 1| - \ln|x + 1| + C \end{aligned}$$

20.

$$\int \frac{x^2}{(x - 3)(x + 2)^2} dx = \int \frac{A}{x - 3} + \frac{B}{x + 2} + \frac{C}{(x + 2)^2} dx$$

So,

$$\frac{x^2}{(x - 3)(x + 2)^2} = \frac{A}{x - 3} + \frac{B}{x + 2} + \frac{C}{(x + 2)^2}$$

Thus,

$$\begin{aligned}x^2 &= A(x+2)^2 + B(x-3)(x+2) + C(x-3) \\x^2 &= (A+B)x^2 + (4A-B+C)x + (4A-6B-3C)\end{aligned}$$

So,

$$\begin{aligned}\int \frac{x^2}{(x-3)(x+2)^2} dx &= \int \frac{-3/5}{x-3} + \frac{8/5}{x+2} + \frac{4}{(x+2)^2} dx \\&= \frac{-3}{5} \ln|x-3| + \frac{8}{5} \ln|x+2| - \frac{4}{x+2} + C\end{aligned}$$

24.

$$\int \frac{x^3}{(x+1)^3} dx = \int 1 - \frac{3x^2+3x+1}{(x+1)^3} dx$$

Now,

$$\frac{3x^2+3x+1}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

So,

$$\begin{aligned}3x^2+3x+1 &= A(x+1)^2 + B(x+1) + C \\&= Ax^2 + (2A+B)x + (A+B+C)\end{aligned}$$

Thus,  $A = 3$ ,  $B = -3$  and  $C = 1$ . So,

$$\begin{aligned}\int \frac{3x^2+3x+1}{(x+1)^3} dx &= \int \frac{3}{x+1} + \frac{-3}{(x+1)^2} + \frac{1}{(x+1)^3} \\&= 3 \ln|x+1| + 3 \frac{1}{x+1} - \frac{1}{2(x+1)^2} + C\end{aligned}$$

28.

$$\begin{aligned}&= \int \frac{x^2-2x-1}{(x-1)^2(x^2+1)} dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1} dx \\&= \frac{x^2-2x-1}{(x-1)^2(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}\end{aligned}$$

So,

$$\begin{aligned}x^2-2x-1 &= A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x-1)^2 \\&= (A+C)x^3 + (-A+B-2C+D)x^2 + (A-C+2D)x + (-A+B+D)\end{aligned}$$

So,

$$\begin{aligned}&= \int \frac{x^2-2x-1}{(x-1)^2(x^2+1)} dx = \int \frac{1}{x-1} + \frac{0}{(x-1)^2} + \frac{-x+0}{x^2+1} dx \\&= \ln|x-1| + -1/2 \ln|x^2+1| + C\end{aligned}$$

38.

$$\int \frac{x^4+1}{x(x^2+1)^2} dx = \int \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

So,

$$\begin{aligned}x^4 + 1 &= A(x^2 + 1)^2 + (Bx + C)x(x^2 + 1) + (Dx + E)x \\ &= (A + B)x^4 + Cx^3 + (2A + B + D)x^2 + (C + E)x + A\end{aligned}$$

Thus,  $A = 1$ ,  $B = 0$ ,  $C = 0$ ,  $D = -2$ , and  $E = 0$ . So,

$$\begin{aligned}\int \frac{x^4 + 1}{x(x^2 + 1)^2} dx &= \int \frac{1}{x} + \frac{-2x}{(x^2 + 1)^2} dx \\ &= \ln |x| - \frac{1}{2(x^2 + 1)} + C\end{aligned}$$