HW §7.2 Numbers 6,10,16,18,24,34,38,46

6.
$$\int \sin^3(mx)dx = \int (1-\cos^2(mx))\sin(mx)dx$$

Let $u = mx$. Then, $du = mdx$ and $\frac{1}{m}du = dx$. Thus,

$$= \frac{1}{m}\int (1-\cos^2(u))\sin(u)du$$

Letting $w = \cos u$, we have $dw = -\sin u$. Thus,

$$= \frac{-1}{m}\int (1-w^2)du$$

$$= \frac{-1}{m}(w - \frac{1}{3}w^3) + C$$

Unsubstituting, we get:

$$= \frac{-1}{m}(\cos u - \frac{1}{3}\cos^3(u)) + C$$

and

$$= \frac{-1}{m}(\cos mx - \frac{1}{3}\cos^3(mx)) + C$$

10.
$$\int \cos^6\theta d\theta = \int (\cos^2\theta)^4 d\theta$$

$$= \int (\frac{1+\cos 2\theta}{2})^3 d\theta$$

$$= \int [\frac{1}{8}\{1 + 3\cos 2\theta + 3\cos^2 2\theta + \cos^3 2\theta\}]d\theta$$

$$= \frac{1}{8}\int \{1 + 3\cos 2\theta + 3\cos^2 2\theta + \cos^3 2\theta\}d\theta$$

$$= \frac{1}{8}\{\theta + \frac{3}{2}\sin 2\theta + 3\int \cos^2 2\theta d\theta + \int (1-\sin^2 2\theta)\cos 2\theta d\theta\}$$

$$= \frac{1}{8}\{\theta + \frac{3}{2}\sin 2\theta + \frac{3}{2}\int (1+\cos 4\theta)d\theta + (\theta - \frac{1}{2}\sin^3 2\theta)\}$$

$$= \frac{1}{8}\{\theta + \frac{3}{2}\sin 2\theta + \frac{3}{2}(\theta + \frac{1}{4}\sin 4\theta) + (\theta - \frac{1}{2}\sin^3 2\theta)\}$$

16. Let $u = \sin\theta$. Then, $du = \cos\theta$. Thus,

$$\int \cos\theta\cos^5(\sin\theta)d\theta = \int \cos^5 u du$$

$$= \int (1-\sin^2 u)^2 \cos u du$$

Letting $w = \sin u$, we get $dw = \cos u$.

$$= \int (1 - w^2)^2 dw$$

$$= w - \frac{1}{3}w^3 + \frac{1}{5}w^5 + C$$

Unsubstituting,

$$= \cos u - \frac{1}{3}\cos^3 u + \frac{1}{5}\cos^5 u + C$$

Unsubstituting for u,

$$= \cos \sin\theta - \frac{1}{3}\cos^3(\sin\theta) + \frac{1}{5}\cos^5(\sin\theta) + C$$

18.

$$\int \cot^5\theta \sin^4\theta d\theta = \frac{\cos^5\theta}{\sin^5\theta}\sin^4\theta d\theta$$

$$= \int \frac{\cos^5\theta}{\sin\theta} d\theta$$

$$= \int \frac{(1 - \sin^2\theta)^2}{\sin\theta}\cos\theta d\theta$$

$$= \int \cot\theta - 2\sin\theta\cos\theta + \sin^3\theta\cos\theta d\theta$$

$$= \ln|\sin\theta| - \sin^2\theta + \frac{1}{4}\sin^4\theta + C$$

24.

$$\int \tan^4 x dx = \int \tan^2 x(\sec^2 x - 1)dx$$

$$= \int \tan^2 x \sec^2 x dx - \int \tan^2 x dx$$

In the first integral, let $u = \tan^x$. Then, $du = \sec^2 x$. We have, then,

$$= \int u^2 du - \int (\sec^2 x - 1)dx$$

$$= \frac{1}{3}u^3 - \tan x + x + C$$

$$= \frac{1}{3}\tan^3 x - \tan x + x + c$$

34.

$$\int \tan^2 x \sec x dx = \int (\sec^2 x - 1)\sec x dx$$

$$= \int \sec^3 x - \sec x dx$$

$$= \int \sec^2 x \sec x dx - \ln|\sec x + \tan x|$$

Let $u = \sec x$ and $dv = \sec^2 x$. Then, by the integration by parts formula:

$$= \sec x \tan x - \int \tan^2 x \sec x dx - \ln|\sec x + \tan x|$$

Thus,

$$\int \tan^2 x \sec x dx = \sec x \tan x - \int \tan^2 x \sec x dx - \ln|\sec x + \tan x|$$

$$\int \tan^2 x \sec x dx = \frac{1}{2}(\sec x \tan x - \ln|\sec x + \tan x|)$$

36.

$$\int \csc^4 x \cot^6 x dx = \int \cot^6 (\cot^2 x + 1) \csc^2 x dx$$

Letting $u = \cot x$, we have $du = -\csc^2 x dx$. So

$$= -\int u^6(u^2+1)du$$

$$= \frac{-1}{9}u^9 - \frac{1}{7}u^7 + C$$

$$= \frac{-1}{9}\cot^9 x - \frac{1}{7}\cot^7 x + C$$

46.

$$\int \frac{dx}{\cos x - 1} = \int \frac{dx}{\cos x - 1}\frac{-\cos x - 1}{-\cos x - 1}dx$$

$$= \int \frac{-\cos x - 1}{1 - \cos^2 x}dx$$

$$= \int \frac{-\cos x - 1}{\sin^2 x}dx$$

$$= \int \frac{-\cos x}{\sin^2 x}dx - \int -\csc^2 x dx$$

$$= \csc x - \cot x + C$$