HW §11.7 Numbers 2,8,14,20,26,32,38

2.
Since
$$\lim_{n\to\infty} \frac{n(n-1)}{n^2+n} = 1$$
and
$$\sum_{n=1}^{\infty} \frac{1}{n}$$
diverges, so too does
$$\sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$$
by the limit comparison test.

8.
Since
$$\lim_{k\to\infty} \frac{2^{k+1}(k+1)!(k+2)!}{2^k k!(k+3)!}$$
$$= \lim_{k\to\infty} \frac{2(k+1)}{(k+3)} = 2$$
the series
$$\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!}$$
diverges by the ratio test.

14.
Since
$$\lim_{n\to\infty} \sin n \neq 0$$
The series
$$\sum_{n=1}^{\infty} \sin n$$
diverges by the test for divergence.

20.
Since
$$\lim_{k\to\infty} \frac{(k+6)5^k}{5^{k+1}(k+5)} = \frac{1}{5}$$
the series
$$\sum_{k=1}^{\infty} \frac{k+5}{5^k}$$

converges by the ratio test.

26.
Since

$$\lim_{n\to\infty} \frac{((n+1)^2+1)5^n}{(n^2+1)5^{n+1}} = \frac{1}{5}$$

The series

$$\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$$

is convergent by the ratio test.

32.
Since

$$\lim_{n\to\infty} \sqrt[n]{\frac{(2n)^n}{n^{2n}}}$$

$$= \lim_{n\to\infty} \frac{2n}{n^2} = 0$$

the series

$$\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$$

converges by the root test.

38.
Since

$$0 \leq \sqrt[n]{2} - 1 \leq (\sqrt[n]{2} - 1)^n$$

for all $n \geq 1$, and since

$$\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)^n$$

converges by the root test (see number 37.)

$$\sum_{n=1}^{\infty} \sqrt[n]{2} - 1$$

converges by the comparison test.