HW §11.6 Numbers 6-24 even

6.
Since

$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$

converges (by the integral test, e.g.)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^4}$$

is absolutely convergent.

8.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

diverges by the limit comparison test with

$$\sum_{n=1}^{\infty} \frac{1}{n}.$$

On the other hand,

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{n^2+1}$$

converges by the alternating series test. So,

$$\sum_{n=1}^{\infty} \frac{-1^{n-1} n}{n^2+1}$$

is conditionally convergent.

10.
Since

$$\lim_{n\to\infty} \frac{e^{-n-1}(n+1)!}{e^{-n} n!} = \infty$$

The series

$$\sum_{n=1}^{\infty} e^{-n} n!$$

diverges by ratio test.

12.
Since

$$\frac{|\sin n|}{4^n} \leq \frac{1}{4^n}$$

and since

$$\sum_{n=1}^{\infty} \frac{1}{4^n}$$

converges as it is a geometric series with $r = 1/4 < 1$,

$$\sum_{n=1}^{\infty} \frac{\sin n}{4^n}$$

converges absolutely.

14.
Since

$$\lim_{n\to\infty} \frac{(n+1)^2 2^{n+1} n!}{n^2 2^n (n+1)!} = 0$$

The series

$$\sum_{n=1}^{\infty} \frac{n^2 2^n}{n!}$$

is absolutely convergent by the ratio test.

16.
Since

$$\sum_{n=1}^{\infty} \frac{3-\cos n}{n^{2/3}-2} \geq \sum_{n=1}^{\infty} \frac{2}{n^{2/3}-2}$$

and

$$\sum_{n=1}^{\infty} \frac{2}{n^{2/3}-2}$$

diverges by the limit comparison test with

$$\sum_{n=1}^{\infty} \frac{2}{n^{2/3}}$$

$$\sum_{n=1}^{\infty} \frac{3-\cos n}{n^{2/3}-2}$$

diverges.

18.
Since

$$\lim_{n\to\infty} \frac{(n+1)!n^n}{n!(n+1)^{n+1}}$$

$$= \lim_{n\to\infty} \frac{(n+1)n^n}{(n+1)^{n+1}}$$

$$= \lim_{n\to\infty} \frac{n^n}{(n+1)^n}$$

$$= \lim_{n\to\infty} (1 - 1/n)^n$$

$$= e^{-1} < 1$$

(use ln to evaluate the limit in the second to last step - as in cal I) The series

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

is absolutely convergent by the ratio test.

20.
Since

$$\lim_{n\to\infty} \sqrt[n]{\frac{1}{(\ln n)^n}} = 0$$

the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$$

is absolutely convergent by the root test.

22.
On one hand

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

diverges by the integral test. On the other hand,

$$\lim_{n\to\infty} \frac{1}{n \ln n}$$

and

$$\frac{1}{n \ln n} = 0$$

is a decreasing sequence,

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n} = 0$$

is conditionally convergent.

24.
Since

$$\lim_{n\to\infty} \sqrt[n]{\frac{1}{(\tan^{-1} n)^n}} = 2/\pi < 1$$

the series

$$\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(\tan^{-1} n)^n}$$

is absolutely convergent by the root test.