HW §11.4 Numbers 4,6,16,18,20

4.
Since
$$\frac{2}{n^3+4} \leq \frac{2}{n^3}$$
for every $n \geq 1$, and since
$$\sum_{n=1}^{\infty} \frac{2}{n^3}$$
converges by the integral test,
$$\sum_{n=1}^{\infty} \frac{2}{n^3+4}$$
converges by the comparison test.

6.
Since
$$\frac{1}{n-\sqrt{n}} \geq \frac{1}{n}$$
for every $n \geq 1$ and since
$$\sum_{n=1}^{\infty} \frac{1}{n}$$
diverges by the integral test,
$$\sum_{n=1}^{\infty} \frac{1}{n-\sqrt{n}}$$
diverges by the comparison test.

16.
Since
$$\frac{1}{\sqrt{n^3+1}} \leq \frac{1}{n^{3/2}}$$
for every $n \geq 1$, and since
$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$
converges by the integral test,
$$\sum_{n=1}^{\infty} \sqrt{n^3+1}$$
converges by the comparison test.

18.
Since

$$\frac{1}{2n+3} \geq \frac{1}{2n}$$

for every $n \geq 1$, and since

$$\sum_{n=1}^{\infty} \frac{1}{2n}$$

diverges by the integral test,

$$\sum_{n=1}^{\infty} 2n+3$$

diverges by the comparison test.

20.
Since

$$\lim_{n\to\infty} \frac{\frac{1+2^n}{1+3^n}}{\frac{2^n}{3^n}} = 1$$

(use L'Hospital and the fact that $d/dx b^x = \ln b b^x$) and since

$$\sum_{n=1}^{\infty} \frac{2^n}{3^n} = \frac{2/3}{1-2/3} = 2$$

$$\sum_{n=1}^{\infty} \frac{1+2^n}{1+3^n}$$

converges by the limit comparison test.