HW §11.3 Numbers 2,4,16,18,20,28

2.

$$\sum_{i=1}^{i=5} a_i \leq \int_1^6 f(x)dx \leq \sum_{i=2}^{i=6} a_i$$

4.

$$\int_1^\infty \frac{1}{\sqrt[4]{x}}dx$$

$$= \lim_{t\to\infty} \int_1^t \frac{1}{\sqrt[4]{x}}dx$$

$$= \lim_{t\to\infty} \frac{4\sqrt[4]{x^3}}{3}|_1^t$$

$$= \lim_{t\to\infty} \frac{4\sqrt[4]{t^3}}{3} - \frac{4}{3}$$

$$= \infty$$

Thus,

$$\sum_1^\infty \frac{1}{\sqrt[4]{n}}$$

diverges. 16.

$$\int_1^\infty \frac{3x+2}{x(x+1)}dx = \lim_{t\to\infty} \int_1^t \frac{3x+2}{x(x+1)}dx$$

$$= \lim_{t\to\infty} \frac{3x+2}{x(x+1)}|_1^t$$

$$= \lim_{t\to\infty} \ln x + 2\ln(x+1)|_1^t$$

$$= \lim_{t\to\infty} \ln t + 2\ln(t+1) - \ln 2$$

$$= \infty$$

Thus, the series

$$\sum_{n=1}^\infty \frac{3n+2}{n(n+1)}dx$$

diverges.

18.

$$\int_1^\infty \frac{1}{x^2-4x+5}dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^2-4x+5}dx$$

$$= \lim_{t\to\infty} \tan^{-1}(x-2)|_1^t$$

$$= \lim_{t\to\infty} \tan^{-1}(t-2) - \tan^{-1}(-1)$$

$$= \pi/2 - \pi/4 = \pi/4$$

Thus, the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2 - 4n + 5} dx$$

converges.

20.

$$\int_1^{\infty} \frac{\ln x}{x^2} dx = \lim_{t \to \infty} \int_1^t \frac{\ln x}{x^2} dx$$
$$= \lim_{t \to \infty} \frac{-\ln x}{x} - \frac{1}{x}|_1^t$$
$$= \lim_{t \to \infty} \frac{-\ln t}{t} - \frac{1}{t} + \frac{1}{1}$$
$$= 1$$

Thus, the series

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^2} dx$$

converges.

28. We cover the case $p \neq 1$ first:

$$\int_1^{\infty} \frac{\ln x}{x^p} dx = \lim_{t \to \infty} \int_1^t \frac{\ln x}{x^p} dx$$
$$= \lim_{t \to \infty} \frac{-\ln x}{(1-p)x^{p-1}} - \frac{1}{(1-p)^2} x^{1-p}|_1^t$$
$$= \lim_{t \to \infty} \frac{-\ln t}{(1-p)t^{p-1}} - \frac{1}{(1-p)^2} t^{p-1} - \frac{-\ln 1}{(1-p)1^{p-1}} - \frac{1}{(1-p)^2} 1^{1-p}$$

which converges if $p > 1$ and diverges if $p < 1$. Thus, the series

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^p} dx$$

converges if $p > 1$ and diverges if $p < 1$.
Now, we consider the case where $p = 1$:

$$\int_1^{\infty} \frac{\ln x}{x} dx = \lim_{t \to \infty} \int_1^t \frac{\ln x}{x} dx$$
$$\lim_{t \to \infty} \frac{(\ln x)^2}{2}|_1^t$$
$$\lim_{t \to \infty} \frac{(\ln t)^2}{2}$$
$$= \infty$$

Thus, when $p = 1$, the series diverges. So, all together, as long as $p < 1$ the series converges and it diverges otherwise.