HW §11.10 Numbers 4-10 even, 16,18,24,26,30

4.

Since

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\sin 2x = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!}$$

6.

Since,

$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

$$\ln(1+x) = \ln(1-(-x)) = -\sum_{n=1}^{\infty} \frac{(-x)^n}{n}$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-x)^{n+1}}{n}$$

8.

Since,

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$xe^x = x\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$$

10.

Recall (p. 252 of the text) that if $f(x) = \cosh x$, then $f'(x) = \sinh x$. Also, by definition, $f(0) = 1$ and $f'(0) = 0$. Thus, we have that $f^{(n)}(0) = 1$ when n is even and $f^{(n)}(0) = 0$ when n is odd. Thus,

$$\cosh x = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n}$$

16.

If $f(x) = \cos x$, then we have

1. $f(\pi) = -1$
2. $f'(\pi) = 0$

3. $f^{(2)}(\pi) = 1$
4. $f^{(3)}(\pi) = -0$

after which the above list repeats. Thus, the Taylor series for $f(x) = \cos x$ centered at $a = \pi$ is:

$$\sum_{n=0}^{\infty}(-1)^{n+1}\frac{1}{(2n)!}x^{2n}$$

18.
If $f(x) = x^{-2}$, then we have

1. $f(1) = 1$
2. $f'(1) = -1/2$
3. $f^{(2)}(1) = 1/6$
4. $f^{(3)}(1) = -1/24$

after which the above list continues according the pattern $f^{(n)} = (-1)^n/(n+1)!$. Thus, the Taylor series for $f(x) = x^{-2}$ centered at $a = \pi$ is:

$$\sum_{n=0}^{\infty}(-1)^n\frac{1}{(n+1)!n!}x^n$$

24.
Since,

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$e^{-x/2} = x\sum_{n=0}^{\infty}\frac{(-x/2)^n}{n!}$$

$$e^x = \sum_{n=0}^{\infty}(-1)^n\frac{x^n}{2^n n!}$$

26.
Since

$$\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$$

$$\sin x^4 = \sum_{n=0}^{\infty}(-1)^n\frac{(x^4)^{2n+1}}{(2n+1)!}$$

$$\sin x^4 = \sum_{n=0}^{\infty}(-1)^n\frac{(x)^{8n+4}}{(2n+1)!}$$

30.
Using the identity $\cos^2 x = \frac{1+\cos 2x}{2}$ we see that

$$\cos^2 x = \frac{1}{2} + \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n}}{(2n)!}$$

which can be re-written in any number of ways.